

DEVELOPMENT OF ANALYTICAL MODELS FOR FORMING AXIS SYMMETRIC DRAWING

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Abstract: It is developed an analytical model of the process of forming an axially symmetric drawing in an interdependent and changing the thickness of the material and strain hardening. Established stress and strain fields at the initial stage of deformation. We obtain the analytical dependence for the size of the plastic region and limit the degree of deformation.

Keywords: stress, strain, deep drawing, hardening.

1. Introduction.

One of the most common technological processes of manufacturing thin-walled axis symmetric shells from sheet metal is deep drawing. Drawing is manufactured as finished products and semi-finished products for further forming of shells of various configuration by methods of pressing, flanging and expanding, as well as a combination of these methods. Among the processes of forming sheet metal axially symmetric drawing is the most studied, which greatly contributed to the widespread use of this process in production. Extensive theoretical studies experimentally studied the influence of various technological factors that have accumulated and systematized a huge production experience. However, a wide range of issues related to the creation of the theory of axially symmetric drawing, quite accurately describing the processes occurring in forming, till now not been resolved. Analysis of factors affecting the process of forming at axis symmetric drawing shows that some factors depend on the state of the raw material (the anisotropy of mechanical properties), the other - on the level of improvement of the process (by the friction clamp and curved edges deforming tools), while others are caused by the process forming (reducing the size of the plastic region, thickness change of the material and the strain hardening). Taking into account that the contact pressure and bending strain on the radial edges of the deforming tools are small [1,2], it is permissible simplify the analysis of the main process in order to assess ability of the material drawing. It is therefore advisable correct statement of axially symmetric drawing and forming the development of an analytical model in an interdependent and changing the thickness of the material and strain hardening. In the present paper does not aim to survey papers on various aspects of the axially symmetric drawing, note that the existing analytical approaches to the creation of evidence-based methods for calculating the parameters of forming of axis symmetric shells under large plastic deformations are constructed, usually based on simplifying assumptions, the main ones being the adoption of a plane schemes deformed state, the analysis methods of deformation theory of plasticity, or the adoption of an ideal rigid-plastic model of a deformable material. Confined solutions derived from such approaches are only approximate and can get extremely rough idea of the stress-strain state of a deformable material [1-4]. The described state due to the fact that the solution of problems in the case of large plastic deformations on the basis of the theory of plasticity with the simultaneous influence of many factors associated with considerable mathematical difficulties that do not allow to obtain closed analytical solutions. However, it is obvious that the analytical solutions are valuable because they allow not only reveal the mechanics of the process of forming, but also to create the conditions for a conscious process control [1]. In the present work, which is a continuation of [5,6], sets the following tasks:

- The development of an analytical model for forming axis symmetric drawing with the interdependent changes the material thickness and strain hardening;
 - The establishment of stress and strain fields, the definition of the size of the plastic region and limiting draw ratio.
- ## 2. Fundamentals of analytical modeling of axially symmetric drawing and the initial equations.

Consider elements deforming a circular blank with a radius R_0 and thickness S_0 in the die with a radius r_0 . Choose the surface of blank

in the middle rectangular coordinate system with axis Z extending in the thickness direction, ρ axis extending in the radial direction and the axis θ , extending in the circumferential direction and perpendicular to axes ρ and Z . At the initial stage of plastic deformation region is a thin-walled shell, with double curvature of variable thickness, which allowed for the use of the membrane theory of shells [1,3]. The initial equations of the theory of plastic flow in the axial symmetry of deformation in conditions of the generalized plane stress state are given in [5,6]. To account for the hardening of the relationship between the yield stress and the current value of equivalent strain is conveniently represented as a power function [2-4] - $\sigma_s = A\varepsilon^n$, where A and n - hardening parameters that depend on the mechanical properties of the metal sheet as follows $A = \sigma_b e^{n \ln(1+\delta)}$ (σ_b, δ - accordingly, the stress of tensile strength and relative elongation at uniform linear tensile). Difficulties in determining the stress and strain fields in the plastic region lies in the fact that some of the plastic state equation set the relationship between stress and the coordinate of the element, others - the stress-increments (speed) strain, and the third-between yield stress and the equivalent strain. The efforts of most researchers [1-4] have focused on bringing these equations to a single structure, which establishes the relationship between stress and the coordinate of the element. In this case, the problem was solved in some cases, the method of successive approximations, when based on the assumption of a plane strain state scheme ($ds = 0$) and perfectly rigid-plastic model of a deformable material ($\sigma_s = \text{const}$) determine the stress field, and then, on the known distribution of stresses in the framework of the deformation theory of plasticity is determined by the strain field. In another approach, the stress field is established on the basis of volume strain state and the assumption of perfectly rigid-plastic model of a deformable material. In both cases, eliminated the possibility of obtaining analytical solutions in an interdependent and changing the thickness of the material and strain hardening. In [5,6] equation of the plastic state are reduced to a single structure through their representation on the plane in a two-dimensional orthogonal coordinate system. Ability to view the stress - strain state in the plane follows from the condition that the amount for which an incompressible material for three main strains are interdependent and have only two degrees of freedom, and of making assumptions about what is happening in forming the conditions of plane stress state [1-4]. Dependence for the radial and circumferential stresses in plane stress which satisfy von Mises yield criterion for this plane are:

$$\sigma_\rho = \sigma_s \frac{2}{\sqrt{3}} \cos\left(\varphi + \frac{\pi}{6}\right), \quad \sigma_\theta = -\sigma_s \frac{2}{\sqrt{3}} \sin \varphi, \quad (1)$$

Dependence (1) appear in the plane considered as the radius $\sigma_s/\sqrt{3}$ of the circles whose centers are located, respectively, $\varepsilon_\rho = 0$ and $\varepsilon_\theta = 0$ the lines and pass through the origin, and the strain hardening is the increase of the radii of the circles and moving their centers by an amount $\Delta\sigma_s$ (Figure1).

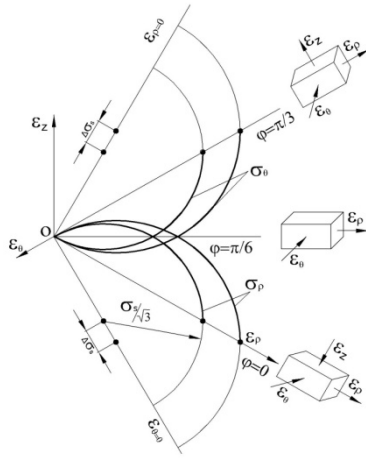


Figure 1. Stress-strain state in an axially symmetric drawing on the plane of strains

The system of equations of plastic state in the plane of strains is converted to a simple proportional relationship between the increments (speed) radial stress and the equivalent strain [5,6]

$$d\sigma_r = \sigma_s d\epsilon. \quad (2)$$

The coefficient of proportionality in (2) is a variable yield stress of the material, which characterizes the strain hardening. Equation (2) is physically justified, as is always the work of uniform strain $dw = \sigma_s d\epsilon$ and stress of drawing are identical [2].

From the definition of an axially symmetric drawing follows that in³ plastic region must satisfy the condition: $\epsilon_\theta \leq 0$, $\sigma_r \cdot \sigma_\theta \leq 0$

that is, all elements of the blank as a result of compressive strain should be reduced in the circumferential direction in bi-axially oppositely stress state. This implies that the parameters φ in (1) can vary $0 \leq \varphi \leq \pi/3$ and take a sector with a central angle equal to $\pi/3$. In this range of the parameter φ vector of the equivalent strain is either parallel or perpendicular to an oblique coordinate axes, resulting in values of the main strains vary from zero to one. At the highest amounts of plastic implements the following scheme of stress-strain states: When $\varphi = 0$, ϵ_r is positive, but ϵ_θ and ϵ_z - negative and numerically equal $\epsilon_r/2$, that is the deformation of

linear expansion is implemented, when $\varphi = \pi/6$, $\epsilon_z = 0$, ϵ_r and ϵ_θ equal in magnitude and opposite in sign, that is, realized pure shear or plane strain in the plane $(\rho; \theta)$, when $\varphi = \pi/3$, ϵ_θ

is negative, but also ϵ_r and ϵ_z positive and equal to $\epsilon_\theta/2$, that is, the deformation of the linear compression is implemented. Therefore, in general, when an axis symmetric drawing, the plastic region is displayed in the plane strain in two sectors to the central characteristic angles equal $\pi/6$ at the boundaries of which vary circuits stress-deformed state. From (1) and (2) if $\varphi = 0$ (line extension), taking into account the law of strain hardening follows the dependence of the parameter of hardening, which coincides with the results of [2-4].

$$\frac{1}{\sigma_s} \cdot \frac{d\sigma_r}{d\epsilon} = \frac{nA\epsilon^{n-1}}{\sigma_s} = \frac{n}{\epsilon} = 1. \quad (3)$$

According to (3), when $\varphi = 0$ the equivalent strain in numerically equal ϵ_r , can not exceed the amount n determined by the test of deformable material to uniaxial tension. The problem of integration of equation (2) taking into account the hardening law is that the intensity of the main deformations increments are not always equal to the differential current of the equivalent strain. It is easy to show that the above equation holds only for a constant ratio of the

principal strains [2]. The assumption of proportional change of principal strains corresponding to the permanence of the relationship $\epsilon_r/\epsilon_\theta$, enables the integration of equation (2). The constant of integration is obtained from the boundary conditions under which the external circuit blank radial stress is zero. Given the boundary conditions, we have

$$\epsilon^{n+1} - \epsilon_0^{n+1} = (1+n)\epsilon^n \frac{2}{\sqrt{3}} \cos(\varphi + \pi/6), \quad (4)$$

where ϵ_0 - equivalent strain of edge member.

To avoid methodological differences is assumed that at the initial stage of deformation value of the deformation of edge member reaches 0,2%, and yield stress $-\sigma_{0,2}$. For this case, the distribution of principal(main) strains are:

$$\begin{aligned} \epsilon_r &= \frac{1+n}{2} \left(1 + \cos 2\varphi - \frac{\sqrt{3}}{3} \sin 2\varphi \right), \\ \epsilon_\theta &= -\frac{1+n}{2} \left(\cos 2\varphi + \frac{\sqrt{3}}{3} \sin 2\varphi \right), \\ \epsilon_z &= -\frac{1+n}{2} \left(1 - \frac{2\sqrt{3}}{3} \sin 2\varphi \right). \end{aligned} \quad (5)$$

Thus, the relation (1), (4) and (5), being an analytical solution of the problem in parametric form, completely determine the stress-strain state of the blank at the initial stage forming in an axially symmetric drawing with the interdependent changing the thickness of the material and strain hardening.

Results and discussion.

In the above analysis of distribution of the main deformations are related to the current deformed state (Euler variables). The summation of the increments at constant strain directions of the principal axes of these elements leads to a logarithmic shell deformations that satisfy the condition of constant volume:

$$\epsilon_r = \ln(d\rho/dr); \quad \epsilon_\theta = \ln(\rho/r); \quad \epsilon_z = \ln(s/s_0)$$

(ρ , s and r , s_0 - respectively, the current the initial coordinates and the thickness of the element).

From the analysis of relationships (5) follow certain characteristics of distribution main strains at the initial stage forming. Consider these features for an ideal rigid-plastic model of a deformable material. Radial plastic deformation at the boundaries of the field is, as in the interval differs significantly from the value of the equivalent strain. The maximum difference when $\varphi \approx 40^\circ$ ($\rho/r \approx 0,69$) in the order to 10%. Circumferential strain near the inner contour of the plastic region has an extreme and when $\varphi = 15^\circ$ ($\rho/r \approx 0,56$) an absolute value reaches a maximum value equal $\sqrt{3}/3$. Deformation of the thickness in the peripheral zone $\pi/6 \leq \varphi \leq \pi/3$ of the blank, namely in the range is positive. Such a result could be expected on the basis of the following arguments. At the initial stage of forming when $\varphi = \pi/6$ ($\rho/r = 0,606$) the radial and circumferential stresses are equal in absolute value, whereby the material thickness remains constant. On the other side edge of blank in a fixed value of deformation edge member tends to zero. Therefore, based on the continuity of the distribution of deformation, it can be assumed that the strain in thickness in this range is positive, and at some point take the extreme value. The marked feature of forming the experimental work presented in [7], however, the authors specified feature left without comment. From (5) it follows that $\varphi = \pi/4$ ($\rho/r \approx 0,75$) the maximum deformation of the thickness attains a value of 0.077, which corresponds to $s/s_0 \approx 1,08$. Consequently, the assertion of different investigators that at the initial stage of forming force of blank-holder is perceived edge element of blank is not well founded.

3.1. Dimensions of the plastic region and the limiting drawing ratio (LDR).

The greatest total radial strain in the output section of the die r_0 is determined from the first equation (5) when $\varphi = 0$, $\varepsilon_{\rho_{\max}} = (1+n)$

. In view of equality $\varepsilon_{\rho_{\max}} = \ln(R_0/r_0)$ we obtain

$$R_0/r_0 = \exp(1+n). \quad (6)$$

From (6) it follows that the maximum size of the plastic region $R_0/r_0 \rightarrow e$ and with the growth of the strain hardening increases exponentially. A similar result was obtained in [2,3]. Significant increase in deformation during drawing as compared to a uniform deformation in uniaxial tension with a physical point of view due to the fact that the different plastic parts of different schemes realized simultaneously stress-deformed state, starting from the linear compression on the external circuit blank to the linear tension of the internal contour. Replacement of schemes stress-strain states is continuous, so that at some intermediate point of pure shear strain is realized. When determining LDR assumed that non-stationary conditions when forming the boundary element of blank deformation in the circumferential direction $\varepsilon_{\theta_{\max}} = |\ln(R_0/r_0)|$ by the maximum radial deformation under linear tension can not exceed n . Substituting these values in (4), after some transformations:

$$\frac{R_0}{r_0} = \exp n^{\frac{n}{n+1}}. \quad (7)$$

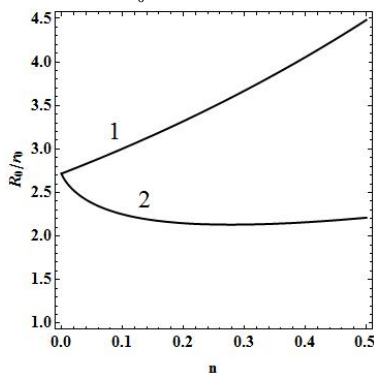


Figure 2 relationship between the size plastic region (1) and LDR (2) of the strain hardening

From the analysis of the relationship (7) that it has a poorly defined extreme. Equating the result of differentiating (7) to zero, we obtain an equation $1+n+\ln n=0$ of the solutions which it follows that

the minimum value $R_0/r_0 \approx 2,131$ of the indicator $n \approx 0,278$. For real structural sheet metal work hardening rate varies within the limits $0,1 \leq n \leq 0,4$. From (7) it follows that while LDR is changing slightly, which served as a pretext for the assertion of the independence of the LDR index n [2,4]. Curve 1 is the boundary of the plastic region in the interval between the curves 1 and 2 realized unfinished drawing, when the outer diameter of the blank, depending on the strain hardening can be reduced only by a certain amount, and the lower curve 2 is implemented deep-drawing process. Based on the relatively small difference between the experimental results and the results obtained on the basis of the developed analytical model, we can assume that the main factors affecting the process of forming in an axially symmetric drawing are the change in material thickness and strain hardening.

4. Conclusions

An analytical model for of forming an axis symmetric drawing interdependent with the change of thickness of the material and the strain hardening, and on this basis set the field of strain and stress in the initial stage of deformation.

It is shown that the size of the largest plastic region realized various schemes of the stress-strain states ranging from uniaxial compression on the outer contour of blank before the uniaxial tension in the internal contour.

Established the size of the plastic region, as well as areas realization of unfinished and deep drawing.

5. References

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